

with respect to the gas phase (infinitely dispersed) is only 12 cal/mole. These differences between the actual behaviour and that of the classical model are due to zero-point energy.†

A similar situation occurs in  $^3\text{He}$  although, since the atomic mass is smaller and the interatomic potential the same, the zero-point energy has an even greater influence here. In addition to the mass difference, the nucleus of  $^3\text{He}$  has a spin value of  $\frac{1}{2}$  while that of  $^4\text{He}$  has no spin. This has important consequences for the relative entropies of the liquid and solid phases of  $^3\text{He}$  at low temperatures and leads to a minimum in the melting curve of  $^3\text{He}$  at about  $0.3^\circ\text{K}$ . In this paper we are concerned with the solid phase region above

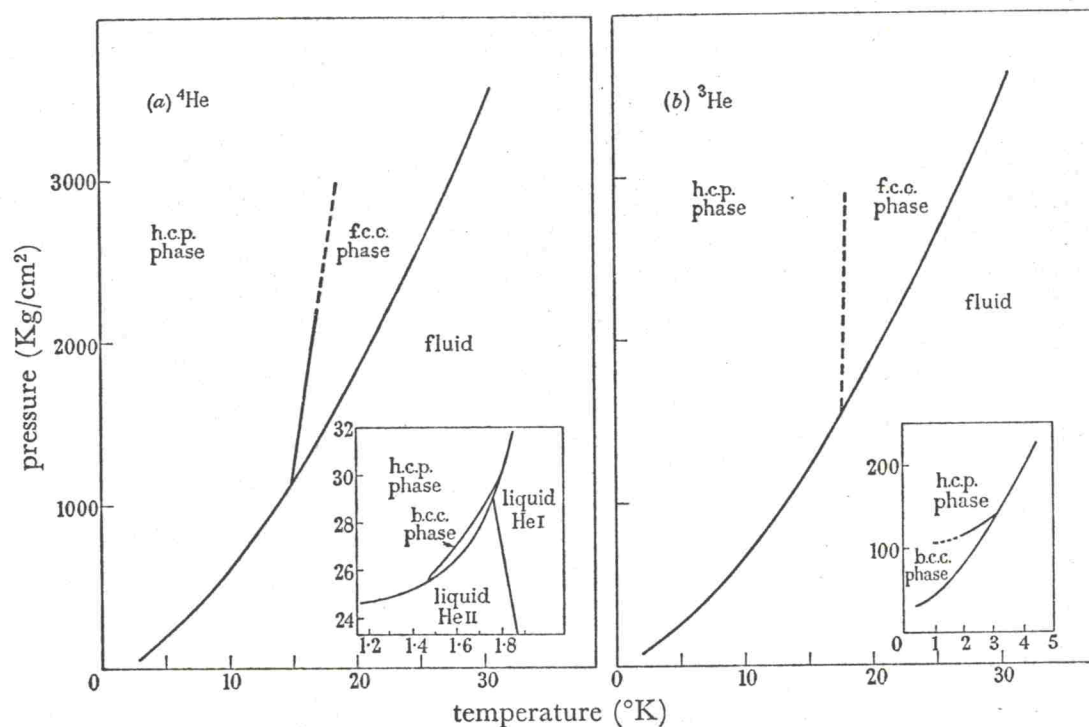


FIGURE 1. (a) Phase diagram of  $^4\text{He}$ . (b) Phase diagram of  $^3\text{He}$ .

about  $3^\circ\text{K}$  and at pressures less than about 2000 atm. In this region, it appears that the nuclear spins are randomly oriented so that there is an additional entropy of  $R \ln 2$  which is absent in  $^4\text{He}$  (cf. the end of § 3.1 below). This spin difference is also associated with a difference in the statistics appropriate to the two isotopes and this shows up very remarkably in the very different properties of the two liquid isotopes. (For a review of the properties of solid and liquid  $^3\text{He}$  see, for example, Bernardes & Brewer 1962.)

In both the solid isotopes of helium there exist at least three different structures, body-centred cubic, hexagonal close-packed and face-centred cubic. The phase diagram for both solid isotopes is shown in figure 1. According to classical lattice theory, a solid composed of atoms which have central, additive short-range forces should have a close-packed structure. In helium where the condition of central, additive short-range forces between atoms should be realized, we might therefore expect to find either of the two close-packed structures: on the other hand, the existence of the b.c.c. structure is unexpected. Almost certainly this is another illustration of the influence of zero-point energy.

† For a review of work on solid  $^4\text{He}$  up to 1956 see, for example, Domb & Dugdale (1957).